Kannisto extrapolation of mortality rates to oldest ages

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1 Kannisto's Model

For high ages (such as 80+) Kannisto's model of age-specific mortality rates is

$$\mu_x = \frac{ae^{bx}}{1 + ae^{bx}} \tag{1}$$

where a and b are parameters that affect the trajectory of mortality rates as age increases. Notice that $\mu_x \to 1$ as $x \to \infty$ regardless of the parameter values. In this model the asymptotic limit of the mortality rate is 1.

The model actually comes from a reparameterized logit function. It can be written as

$$\mu_x = \frac{e^{Y_x}}{1 + e^{Y_x}}$$

or equivalently as

$$logit(\mu_x) = Y_x$$

where $Y_x = \ln(ae^{bx}) = \ln a + bx$.

2 Extrapolating from high to very high ages

If we have data or model estimates for high ages (such as 70–89), we can extrapolate with the Kannisto model to even higher ages (such as 90–120) as follows:

1. Estimate $\hat{a}$ and $\hat{b}$ in one of two ways:
   - calculate $Y_x = logit(\mu_x)$ for the observed ages; regress Y_x on x to get slope and intercept estimates over the range of high ages that are already observed; define $\hat{a} = \exp(\text{intercept})$ and $\hat{b} = \text{slope}$
   - if deaths D_x and exposures N_x are available for the observed ages, estimate the parameters a and b directly from a Poisson regression model with $D_x \sim Pois[N_x\, \mu_x(a,b)]$ with μ_x defined as in Eq (1).
2. extrapolate using $\hat{\mu}_x = \frac{\hat{a}e^{\hat{b}x}}{1+\hat{a}e^{\hat{b}x}}$ from Eq (1) for the very high ages